By definition, F_D is the drag force parallel to v_r, and F_L is the lift force perpendicular to v_r. If F_{axial} is the increment of axial force, and F_{power} that of tangential force (which produces rotor acceleration and power), then

$$F_{axial} = F_L \cos\phi + F_D \sin\phi \tag{9.50}$$

$$F_{power} = F_L \sin\phi + F_D \cos\phi \tag{9.51}$$

Stream tube theory considers a cylinder or tube of the oncoming wind incident on elements of the blades at radius r from the axis, chord (width) $c = c(r)$, and incremental length dr. One such cylinder can be treated independently of others both upstream and downstream of the turbine disc. Advanced texts should be consulted for the further development, e.g. Burton *et al.*, (2000), Hansen (2000).

9.6 Characteristics of the wind

9.6.1 Basic meteorological data and wind speed time series

All countries have national meteorological services that record and publish weather related data, including wind speeds and directions. The methods are well established and co-ordinated within the World Meteorological Organisation in Geneva, with a main aim of providing continuous runs of data for many years. Consequently only the most basic data tend to be recorded at a few permanently staffed stations using robust and trusted equipment. Unfortunately for wind power prediction, measurements of wind speed tend to be measured only at the one standard height of 10 m, and at stations near to airports or towns where shielding from the wind might be a natural feature of the site. Therefore to predict wind power conditions at a specific site, standard meteorological wind data from the nearest station are only useful to provide first order estimates, but are not sufficient for detailed planning. Usually careful measurements around the nominated site are needed at several locations and heights for several months to a year. These detailed measurements can then be related to the standard meteorological data, and these provide a long-term base for comparison. In addition, information is held at specialist wind power data banks that are obtained from aircraft measurements, wind power installations and mathematical modelling, etc. Such organised and accessible information is increasingly available on the Internet. Wind power prediction models (e.g. the propriety WAsP models developed in Denmark) enable detailed wind power prediction for wind turbine prospective sites, even in hilly terrain.

Classification of wind speeds by meteorological offices is linked to the historical Beaufort scale, which itself relates to visual observations. Table 9.1 gives details together with the relationship between various units of wind speed.

A standard meteorological measurement of wind speed measures the 'length' or 'run' of the wind passing a 10 m high cup anemometer in 10 min. Such measurements may be taken hourly, but usually less frequently. Such data give little information about fluctuations in the speed and direction of the wind necessary for accurately predicting wind turbine performance. Continuously reading anemometers are better, but these too will have a finite response time. A typical continuous reading trace, Figure 9.15(a), shows the rapid and random fluctuations that occur. Transformation of such data into the frequency domain gives the range and importance of these variations, Figure 9.15(b).

The direction of the wind refers to the compass bearing *from* which the wind comes. Meteorological data are usually presented as a wind rose, Figure 9.16(a), showing the average speed of the wind within certain ranges of direction. It is also possible to show the distribution of speeds from these directions on a wind rose, Figure 9.16(b). Such information is of great importance when siting a wind machine in hilly country, near buildings, or in arrays of several machines where shielding could occur. Changes in wind direction may be called 'wind shift'; $0.5\,\mathrm{rad\,s^{-1}}$ ($30°\,\mathrm{s^{-1}}$) is a rapid change, e.g. in hilly terrain. Such changes may damage a wind turbine more than an extreme change in wind speed.

9.6.2 Variation with height

Wind speed varies considerably with height above ground; this is referred to as *wind shear*. A machine with a hub height of (say) 30 m above other obstacles will experience far stronger winds than a person at ground level. Figure 9.17 shows the form of wind speed variation with height z in the near-to-ground boundary layer up to about 100 m. At $z = 0$ the air speed is always zero. Within the height of local obstructions wind speed increases erratically, and violent directional fluctuations can occur in strong winds. Above this erratic region, the height/wind speed profile is given by expressions of the form

$$z - d = z_0 \exp(u_z/V) \tag{9.52}$$

Hence

$$u_z = V \ln\left(\frac{z-d}{z_0}\right) \tag{9.53}$$

Here d is the zero plane displacement with magnitude a little less than the height of local obstructions, the term z_0 is called the roughness length and V is a characteristic speed. On Figure 9.17 the function is extrapolated to negative values of u to show the form of the expression. Readers should

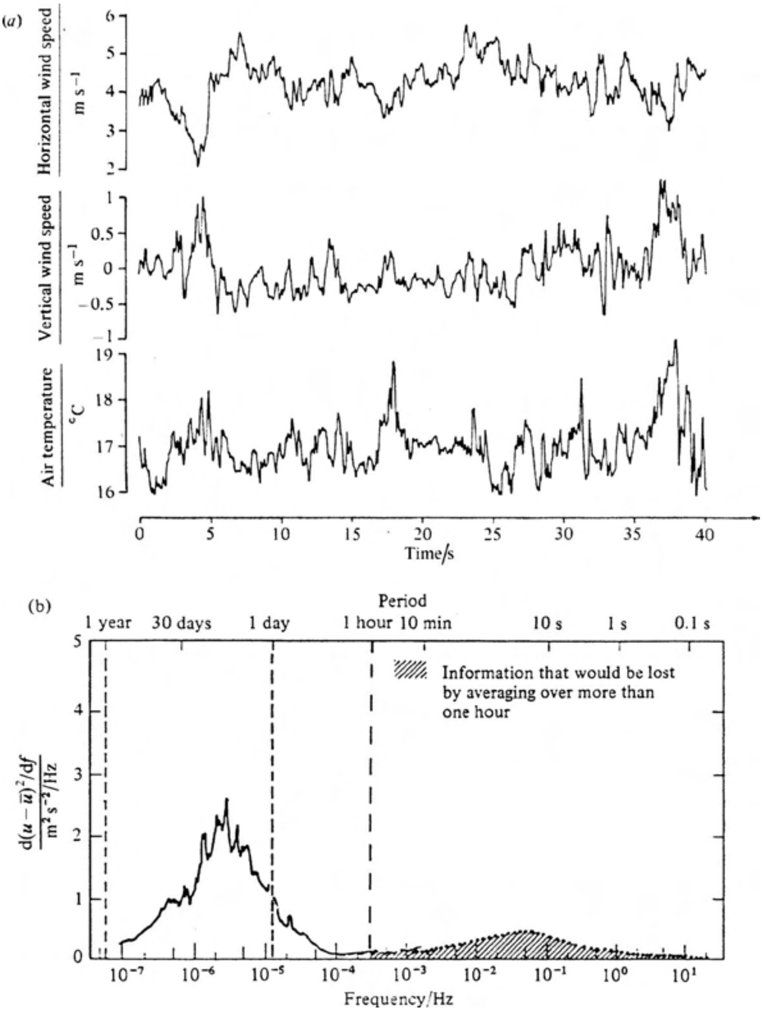

Figure 9.15 (a) Continuous anemometer reading. A short section of a record of horizontal wind speed, vertical wind speed and temperature at a height of 2 m at the meteorological field, Reading University, UK. Note the positive correlations between vertical wind speed and temperature, and the negative correlations between horizontal and vertical wind speeds. (b) Frequency domain variance spectrum [after Petersen (1975)]. The graph is a transformation of many time series measurements in Denmark, which have been used to find the square of the standard deviation (the variance) of the wind speed u from the mean speed $\bar{u}$. Thus the graph relates to the energy in wind speed fluctuations as a function of their frequency; it is sometimes called a 'Van der Hoven' spectrum.

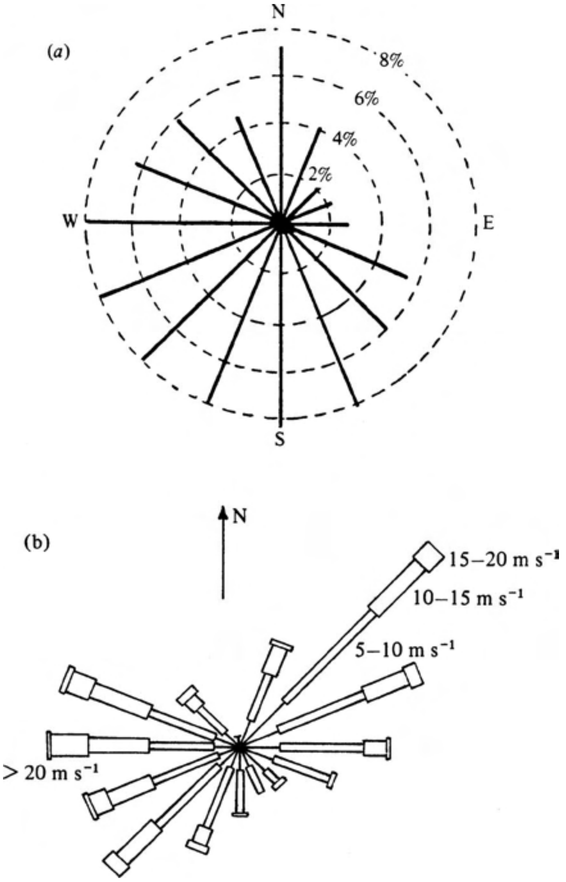

Figure 9.16 Wind rose from accumulated data. (a) Direction. Station on the Scottish island of Tiree in the Outer Hebrides. The radial lines give percentages of the year during which the wind blows from each of 16 directions. The values are ten-year means and refer to an effective height above ground of 13 m. (b) Direction and distribution of speed. Malabar Hill on Lord Howe Island, New South Wales. The thicker sections represent the proportion of time the wind speed is between the specified values, within 16 directional sectors. After Bowden *et al.* (1983).

consult texts on meteorology and micrometeorology for correct detail and understanding of wind speed boundary layer profiles. However, the most important practical aspect is the need to place a turbine well above the height of local obstructions to ensure that the turbine rotor disc receives a strong uniform wind flux across its area without erratic fluctuations.

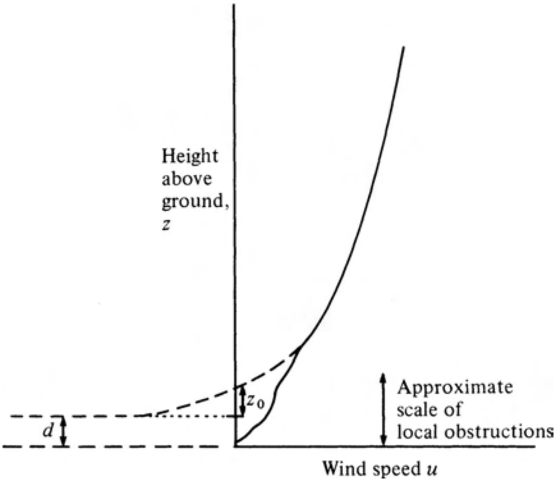

Figure 9.17 Wind speed variation with height; 'wind shear', see (9.52).

The best sites for wind power are at the top of smooth dome-shaped hills that are positioned clear of other hills. In general the wind should be incident across water surfaces or smooth land for several hundred metres, i.e. there should be a good fetch. Most wind turbines operate at hub heights between 5 m (battery chargers) and 100 m (large, grid linked). However, it is common for standard meteorological wind speed measurements u_s to be taken at a height of 10 m. An approximate expression often then used to determine the wind speed u_z at height z is

$$u_z = u_s \left(\frac{z}{10\,\mathrm{m}}\right)^{b'} \tag{9.54}$$

It is often stated that $b' = 1/7 = 0.14$ for open sites in 'non-hilly' country. Good sites should have small values of b' to avoid changes in oncoming wind speed across the turbine disc, and large values of mean wind speed $\overline{u}$ to increase power extraction. Great care should be taken with this formula, especially for $z > 50\,\mathrm{m}$. Problem 9.12 shows that if (9.54) remains realistic, then extremely high towers ($> 100\,\mathrm{m}$, say) are probably unwarranted unless the rotor diameter is very large also.

9.6.3 Wind speed analysis, probability and prediction

Implementation of wind power requires knowledge of future wind speed at the turbine sites. Such information is essential for the design of the machines and the energy systems, and for the economics. The seemingly random nature of wind and the site-specific characteristics makes such information challenging, yet much can be done from statistical analysis, from correlation of measurement time-series and from meteorology. The development of wind power has led to great sophistication in the associated analysis, especially involving data handling techniques and computer modelling. However, Example 9.1 and Table 9.3 illustrate the principles of such analysis, by showing how the power available from the wind at a particular site can be calculated from very basic measured data on the distribution of wind speed at that site. Commercial measurement techniques are more sophisticated, but the principles are the same.

Example 9.1 Wind speed analysis for the island of North Ronaldsay, Orkney

A ten-minute 'run of the wind' anemometer was installed at 10 m height on an open site near a proposed wind turbine. Five readings were recorded each day at 9 a.m., 12 noon, 3 p.m., 6 p.m. and 9 p.m., throughout the year. Table 9.3 gives a selection of the total data and analysis, with columns numbered as below.

Table 9.3 Wind speed analysis for the example of North Ronaldsay. This is a selection of the full data of Barbour (1984), to show the method of calculation. Columns numbered as in Example 9.1

1 u'	2 dN/du	3 Φ_u	4 $\Phi_{u \geq u'}$	5 $\Phi_u u$	6 u^3	7 $\Phi_u u^3$	8 P_u	9 $P_u \Phi_u$
$(\mathrm{m\,s^{-1}})$	$(\mathrm{m\,s^{-1}})^{-1}$	$(\mathrm{m\,s^{-1}})^{-1}$			$(\mathrm{m\,s^{-1}})^3$	$(\mathrm{m\,s^{-1}})^2$	kWm^{-2}	$(\mathrm{W/m^2})/(m/s)$
>26	1	0.000	0.000	0.000	17576	0.0	11.4	0.0
25	1	0.001	0.001	0.025	15625	15.6	10.2	10.2
24	1	0.001	0.002	0.024	13824	20.7	9.0	9.0
23	2	0.002	0.004	0.046	12167	18.3	7.9	15.8
22	4	0.002	0.006	0.044	10648	21.3	6.9	13.8
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
8	160	0.091	0.506	0.728	512	46.6	0.3	27.3
7	175	0.099	0.605	0.693	343	340	0.2	19.8
6	179	0.102	0.707	0.612	216	22.0	0.1	10.2
5	172	0.098	0.805	0.805	125	12.3	0.1	9.8
4	136	0.077	0.882	0.882	64	4.9	0.0	0.0
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
0	12	0.007		0	0		0	0
Totals	1763	1.000		8.171		1044.8		
Comment		Peaks at 6.2 m s^{-1}		$u_m =$ 8.2 m s^{-1}		$(\overline{u^3})^{1/3} =$ 10.1 m s^{-1}		

1 Readings were classed within intervals of $\Delta u = 1\,\mathrm{m\,s^{-1}}$, i.e. 0.0–0.9; 1.0–1.9, etc. A total of $N = 1763$ readings were recorded, with 62 missing owing to random faults.

2 The number of occurrences of readings in each class was counted to give $\Delta N(u)/\Delta u$, with units of number per speed range ($\mathrm{d}N/\mathrm{d}u$ in Table 9.3).

Note: $\Delta N(u)/\Delta u$ *is* a number per speed range, and so is called a frequency distribution of wind speed. Take care, however, to clarify the interval of the speed range Δu (in this case 1 m/s , but often a larger interval).

3 $[\Delta N(u)/\Delta u]/N = \Phi_u$ is a normalized probability function, often called the *probability distribution of wind speed*. Φ_u is plotted against u in Figure 9.18. The unit of Φ_u is the inverse of speed interval, in this case $(1\,\mathrm{m\,s})^{-1}$. $\Phi_u \Delta u$ is the probability that the wind speed is in the class defined by u (i.e. u to $u + \Delta u$). For one year $\Sigma \Phi_u \Delta u = 1$.

4 The cumulative total of the values of $\Phi_u \Delta u$ is tabulated to give the probability, $\Phi_{u>u'}$, of speeds greater than a particular speed u'. The units are number per speed range multiplied by speed ranges i.e. dimensionless. This function is plotted in Figure 9.19, and may be interpreted as the proportion of time in the year that u exceeds u'.

5 The average or mean wind speed u_m is calculated from $u_\mathrm{m} \Sigma \Phi_u = \Sigma \Phi_u u$. The mean speed $u_\mathrm{m} = 8.2\,\mathrm{m\,s^{-1}}$ is indicated on Figure 9.18. Notice that u_m is greater than the most probable wind speed of $6.2\,\mathrm{m\,s^{-1}}$ on this distribution.

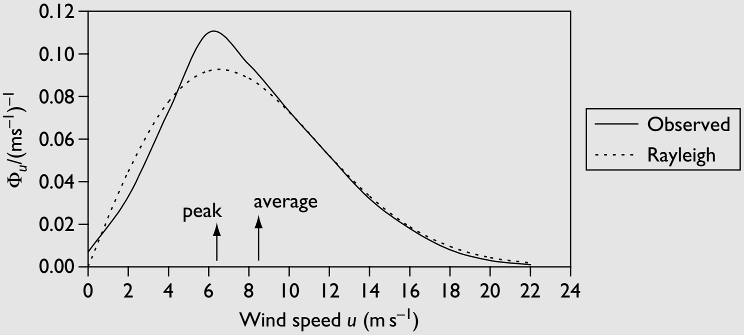

Figure 9.18 Probability distribution of wind speed against wind speed. Data for North Ronaldsay from Barbour. (——) measured data (from Table 9.3) ; (- - -) Rayleigh distribution (9.69) fitted to match mean speed $\bar{u}$. Note that the average wind speed ($8.2\,\mathrm{m\,s^{-1}}$) exceeds the most probable wind speed ($6.2\,\mathrm{m\,s^{-1}}$); see Example 9.1 and (9.75).

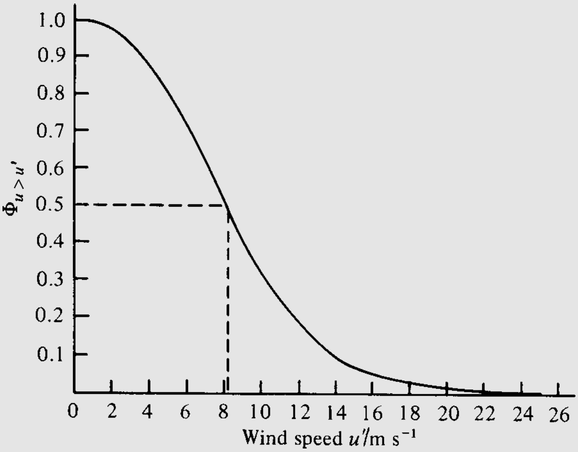

Figure 9.19 Probability of wind speeds greater than a particular speed u', for example of North Ronaldsay.

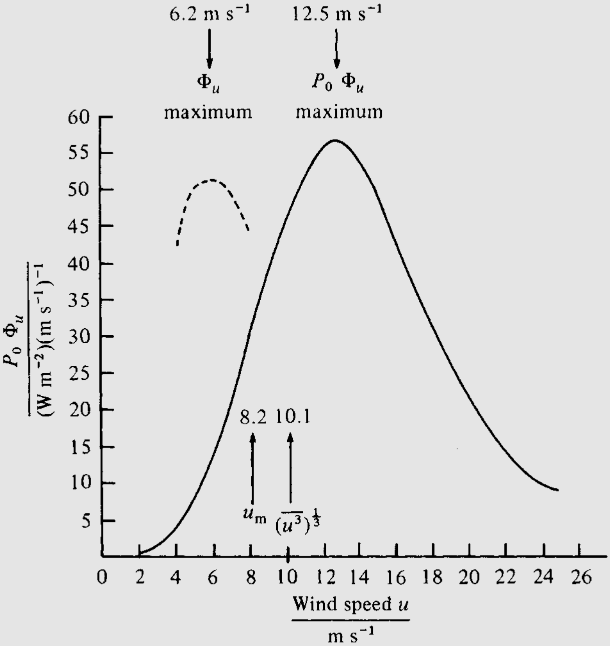

Figure 9.20 Distribution of power in the wind, for example of North Ronaldsay.

6 Values of u^3 are determined.
7 $\Phi_u u^3$ allows the mean value $\overline{u^3}$ to be determined from $(\overline{u^3})\Sigma\Phi_u = \Sigma\Phi_u u^3$, see (9.71). $\overline{u^3}$ relates to the power in the wind.
8 The power per unit area of wind cross-section is $P_0 = \frac{1}{2}\rho u^3$. If $\rho = 1.3\,\mathrm{kg\,m^{-3}}$ then $P_u = Ku^3$ where $K = (0.65 \times 10^{-3})\,\mathrm{W\,m^{-2}(m/s)^{-3}}$.
9 $P_u\Phi_u$ is the distribution of power in the wind, Figure 9.20. Notice that the maximum of $P_u\Phi_u$ occurs on North Ronaldsay at $u = 12.5\,\mathrm{m\,s^{-1}}$, about twice the most probable wind speed of $6.2\,\mathrm{m\,s^{-1}}$.
10 Finally, Figure 9.21 plots the power unit area in the wind at u' against $\Phi_{u>u'}$, to indicate the likelihood of obtaining particular power.

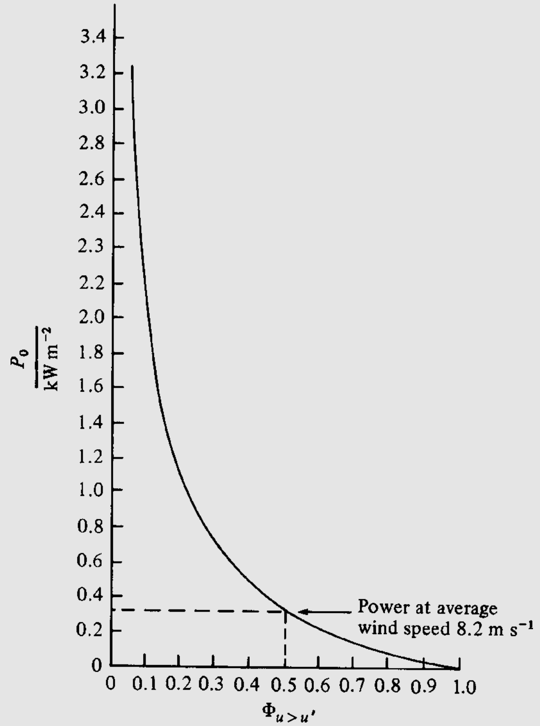

Figure 9.21 Power per unit area in the wind against probability of wind speeds greater than a particular speed u'.

The analysis of Example 9.1 is entirely in terms of the probability of wind characteristics; in essence we have considered a 'frequency domain' analysis and not the 'time domain'. The time domain, including turbulence and gustiness, is considered in Section 9.6.5.

9.6.4 Wind speed probability distributions: Weibull and Rayleigh

The analysis of Example 9.1 depended solely on field data and repetitive numerical calculation. It would be extremely useful if the important function Φ_u, the probability distribution of wind speed, could be given an algebraic form that accurately fitted the data. Two advantages follow: (1) fewer data need be measured, and (2) analytic calculation of wind machine performance could be attempted.

Using the symbols of the previous section,

$$\Phi_{u>u'} = \int_{u=u'}^{\infty} \Phi_u(u)\mathrm{d}u = 1 - \int_0^{u'} \Phi_u \,\mathrm{d}u \tag{9.55}$$

Therefore, by the principles of calculus,

$$\frac{\mathrm{d}\Phi_{u>u'}}{\mathrm{d}u'} = -\Phi_u \tag{9.56}$$

For sites without long periods of zero wind, i.e. the more promising sites for wind-power, usually with $\overline{u} > 5\,\mathrm{m\,s^{-1}}$, usually a two-parameter exponential function can be closely fitted to measure wind speed data. One such function, often used in wind speed analysis, is the *Weibull function* shown in Figure 9.22 obtained from

$$\Phi_{u>u'} = \exp\left[-\left(\frac{u'}{c}\right)^k\right] \tag{9.57}$$

so (Weibull):

$$\Phi_u = \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} \exp\left[-\left(\frac{u}{c}\right)^k\right] \tag{9.58}$$

Figure 9.22 shows the form of $\Phi_{u>u'}$ and Φ_u for different values of k around 2.0. Such curves often give very good fit to experimental data, with k between 1.8 and 2.3 and c near the mean wind speed for the site. See also Figure 9.18, which compares actual data for North Ronaldsay to a Rayleigh distribution, with $k = 2$. The dimensionless parameter k is called the shape factor (see Figure 9.22(a) for the obvious reason), and c, unit $\mathrm{m\,s^{-1}}$, the scale factor. Note that when $\Phi_{u>u'} = 1/e$, $u'/c = 1$ and so c can be obtained as equal to the wind speed measurement at that point.

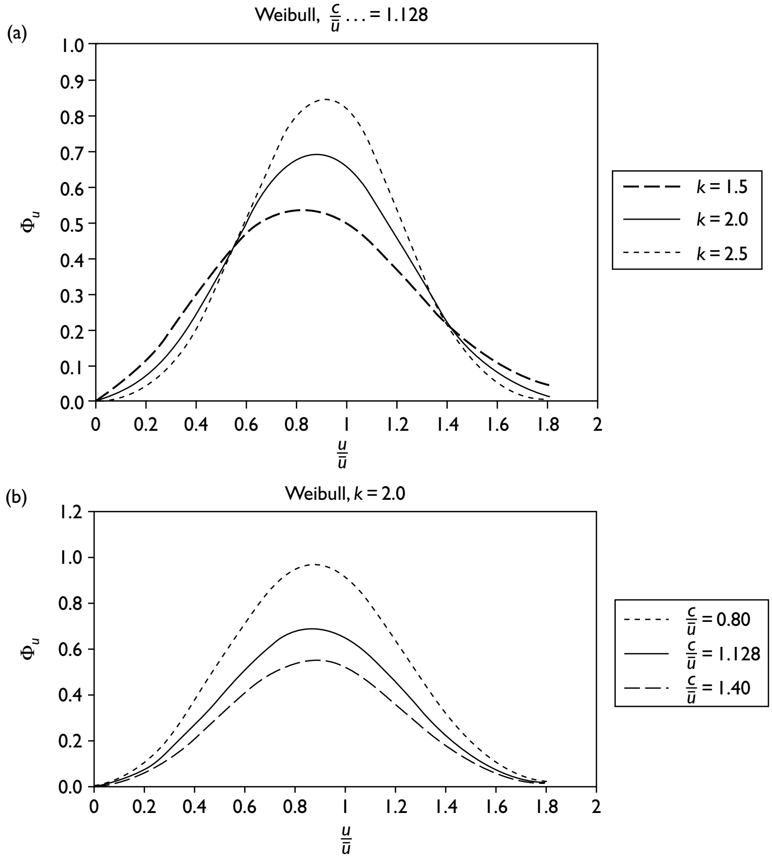

Figure 9.22 Weibull distribution curves. (a) Varying k: curves of Φ_u for $c = 1.128\bar{u}$ and $k = 1.5, 2.0, 2.5$. Curve for $k = 2$, $c/\bar{u} = 1.128$ is the Rayleigh distribution. (b) Varying c: curves of Φ_u for $k = 2$ and $c/\bar{u} = 0.80, 1.128, 1.40$. As probability functions, areas under curves may be normalised to 1.0.

For many sites it is adequate to reduce (9.58) to the one-parameter *Rayleigh distribution* (also called the chi-squared distribution), by setting $k = 2$. So (Rayleigh)

$$\Phi_u = \frac{2u}{c^2} \exp\left[-\left(\frac{u}{c}\right)^2\right] \tag{9.59}$$

For a Rayleigh distribution, $c = 2\bar{u}/\sqrt{\pi}$; see (9.68).

The mean wind speed is then

$$\bar{u} = \frac{\int_0^\infty \Phi_u u \,\mathrm{d}u}{\int_0^\infty \Phi_u \,\mathrm{d}u} \tag{9.60}$$

For the Weibull distribution of (9.58), this becomes

$$\bar{u} = \frac{\int_0^\infty u u^{k-1} \exp\left[-(u/c)^k\right]\mathrm{d}u}{\int_0^\infty u^{k-1} \exp\left[-(u/c)^k\right]\mathrm{d}u} \tag{9.61}$$

Let $(u/c)^k = v$, so $\mathrm{d}v = (k/c^k)u^{k-1}\,\mathrm{d}u$. Equation (9.61) becomes

$$\bar{u} = \frac{c\int_0^\infty v^{t/k} \exp\left[-v\right]\mathrm{d}v}{\int_0^\infty \exp\left[-v\right]\mathrm{d}v} \tag{9.62}$$

The denominator is unity, and the numerator is in the form of a standard integral called the factorial or gamma function, i.e.

$$\Gamma(z+1) = z! = \int_{v=0}^{\infty} v^z e^{-v}\,\mathrm{d}v \tag{9.63}$$

Note that the gamma function is unfortunately usually written, as here, as a function of $z+1$ and not z (refer to Jeffreys and Jeffreys).

Thus

$$\bar{u} = c\Gamma(1+1/k) = c[(1/k)!] \tag{9.64}$$

Using the standard mathematics of the gamma function, the mean value of u^n is calculated, where n is an integer or fractional number, since in general for the Weibull function

$$\overline{u^n} = c^n\Gamma(1+n/k) \tag{9.65}$$

For instance the mean value of u^3 becomes

$$\overline{u^3} = c^3\Gamma(1+3/k) \tag{9.66}$$

from which the mean power in the wind is obtained.

There are several methods to obtain values for c and k for any particular wind distribution (e.g. see Rohatgi and Nelson, or Justus *et al.*). Some examples are:

1 Fit the distribution to meteorological measurements. For instance if $\overline{u}$ and $\overline{u^3}$ are known, then (9.64) and (9.66) are simultaneous equations

with two unknowns. Modern data collection and online analysis methods enable mean values to be continuously accumulated without storing individual records, so $\overline{u}$ and $\overline{u^3}$ are easily measured.

2. Measure $\overline{u}$ and the standard deviation of u about $\overline{u}$, to give $(\overline{u^2} - \overline{u}^2)$ and hence $\overline{u^2}$.
3. Plot the natural log of the natural log of $\Phi_{u>u'}$ in (9.57) against ln u; the slope is k, and hence the intercept gives c.

The Rayleigh distribution is particularly useful for preliminary analysis of wind power potential across a large area, as often the only data that are available are maps showing interpolated curves of mean wind-speed – which is the only parameter needed to fit a Rayleigh distribution.

Example 9.2 Rayleigh distribution analysis

Show that for the Rayleigh distribution:

1. $\Phi_{u>u'} = \exp\left[-\frac{\pi}{4}\left(\frac{u'}{\bar{u}}\right)^2\right]$
2. $(\overline{u^3})^{1/3} = 1.24\bar{u}$
3. Power in the wind per unit area is $\bar{P}_0/A \approx \rho(\bar{u})^3$
4. $\Phi_u(\max)$ occurs at $u = (2/\pi)^{1/2}\bar{u} = 0.80\bar{u}$
5. $(\Phi_u u^3)(\max)$ occurs at $u = 2(2/\pi)^{1/2} = 1.60\bar{u}$

Solution

In (9.62) with $k = 2$,

$$\bar{u} = c\Gamma(1 + 1/2) = c[(1/2)!] \tag{9.67}$$

where by definition

$$(1/2)! = \int_0^\infty u^{1/2} e^{-u} \mathrm{d}u$$

By a standard integral

$$(1/2)! = \sqrt{\pi}/2$$

Hence in (9.67) for the Raleigh distribution,

$$c = 2\bar{u}/\sqrt{\pi} = 1.13\bar{u} \tag{9.68}$$

In (9.59), the Rayleigh distribution becomes

$$\Phi_u = \frac{\pi u}{2\bar{u}^2} \exp\left[-\frac{\pi}{4}\left(\frac{u}{\bar{u}}\right)^2\right] \tag{9.69}$$

and by (9.55)

$$\Phi_{u>u'} = \int_{u=u'}^{\infty} \Phi_u \, \mathrm{d}u = \exp\left[-\frac{\pi}{4}\left(\frac{u'}{\bar{u}}\right)^2\right] \tag{9.70}$$

Also

$$\overline{u^3} = \frac{\int_0^\infty \Phi_u u^3 \mathrm{d}u}{\int_0^\infty \Phi_u \mathrm{d}u} = \left[\frac{\pi}{2\bar{u}^2} \int_0^\infty u^4 \exp -\frac{\pi}{4}\left(\frac{u}{\bar{u}}\right)^2\right] \mathrm{d}u \tag{9.71}$$

By standard integrals of the gamma function this reduces to

$$\overline{u^3} = K(\bar{u})^3 \tag{9.72}$$

where K is called the ‘energy pattern factor’. For the Rayleigh distribution of (9.71), $K = (6/\pi) = 1.91$, see problem 9.5.
Hence

$$(\overline{u^3})^{1/3} = (1.91)^{1/3}\bar{u} = 1.24\bar{u} \tag{9.73}$$

A very useful relationship between mean wind speed and average annual power in the wind per unit area follows:

$$\frac{\overline{P_0}}{A} = \frac{1}{2}\rho\overline{u^3} \approx \rho\,(\bar{u})^3 \tag{9.74}$$

By differentiation to obtain the values of u for maxima in Φ_u and Φ_u^3, and again using the standard integral relationships of the gamma function (see Problem 9.5):

$$\Phi_u(\text{max}) \text{ occurs at } u = (2/\pi)^{1/2}\bar{u} = 0.80\bar{u} \tag{9.75}$$

and

$$\left(\Phi_u u^3\right)(\text{max}) \text{ occurs at } u = 2(2/\pi)^{1/2}\bar{u} = 1.60\bar{u} \tag{9.76}$$

Example 9.3 Rayleigh distribution fitted to measured data
Apply the results of Example 9.2 to the data from North Ronaldsay in Example 9.1.

Solution
For North Ronaldsay $\overline{u} = 8.2\,\mathrm{m\,s^{-1}}$. Therefore by (9.75), $\Phi_u(\max)$ is at $(0.80)(8.2\,\mathrm{m\,s^{-1}}) = 6.6\,\mathrm{m\,s^{-1}}$. The measured value from Figure 9.18 is $6.2\,\mathrm{m\,s^{-1}}$.

By (9.76), $(\Phi_u u^3)(\max)$ is at $(1.60)(8.2\,\mathrm{m\,s^{-1}}) = 13\,\mathrm{m\,s^{-1}}$. The measured value from Figure 9.20 is $12.5\,\mathrm{m\,s^{-1}}$.

By (9.73), $(\overline{u^3})^{1/3} = (1.24)(8.2\,\mathrm{m\,s^{-1}}) = 10.2\,\mathrm{m\,s^{-1}}$. The measured value from Figure 9.20 is $10.1\,\mathrm{m\,s^{-1}}$.
See also Figure 9.18.

9.6.5 Wind speed and direction variation with time

From Figure 9.15, note the importance of fluctuations $\sim$ 10 s. These not only contain significant energy, but lead to damaging stresses on wind machines. A measure of all such time variations is the *turbulence intensity*, equal to the standard deviation of the instantaneous wind speed divided by the mean value of the wind speed. Turbulence intensity is a useful measure over time intervals of a few minutes; values of around 0.1 imply a 'smooth' wind, as over the sea, and values of around 0.25 imply a very gusty, large turbulence, wind, as in mountainous locations. Turbulence can be expected to reduce with height above ground. There are similar expressions for the variation of wind direction with time, sometimes called 'wind shift'.

A wind turbine, especially medium to large size, will not respond quickly enough, or have the aerodynamic properties, to 'follow' rapid changes in wind speed and direction. Therefore energy in wind turbulence and shift may not be captured, but this is an advantage if fatigue damage is thereby lessened.

The more wind turbines and wind farms are dispersed on a grid, the less correlated are the short-term variations and the easier it is to accept increased capacity of wind power. The product of wind speed and the correlation time period is called the *coherence distance*. For short periods, say 10 s of turbulence, the coherence distance will be usually less than the 'length' of a wind farm, so such variations are averaged out. For periods of about 30 min, the correlation distance may be about 20 km; in which case wind farm output dispersed over distances of the order of 100 km will also not correlate, with variations in power not apparent over the whole grid. Only when the coherence distance becomes larger than the scale of the grid are fluctuations not smoothed out by diversity of the site locations.

9.7 Power extraction by a turbine

The fraction of power extracted from the wind by a turbine is, by (9.15), the power coefficient C_P as discussed in Section 9.3. At any instant, C_P *is* most dependent on the tip-speed ratio λ, unless the machine is controlled for other reasons (as happens below the cut-in wind speed and usually above the rated output). The strategy for matching a machine to a particular wind regime will range between the aims of (1) maximizing total energy production in the year (e.g. for fuel saving in a large electricity network), and (2) providing a minimum supply even in light winds (e.g. for water pumping to a cattle trough). In addition secondary equipment, such as generators or pumps, has to be coupled to the turbine, so its power matching response has to be linked to the turbine characteristic. The subject of power extraction is therefore complex, incorporating many factors, and in practice a range of strategies and types of system will be used according to different traditions and needs.

This section considers power extracted by the turbine, which will have a rated power capacity P_R produced at the specified rated wind speed u_R. From Section 9.6.3, Φ_u *is* the probability per wind speed interval that the wind speed will be in the interval u to $(u+\mathrm{d}u)$ (i.e. $\Phi_u\,\mathrm{d}u$ is the probability of wind speed between u and $u+\mathrm{d}u$). Let E be the total energy extracted by the turbine in the period T, and let E_u be the energy extracted per interval of wind speed when the wind speed is u. Then

$$E=\int_{\mathrm{u}=0}^{\infty}E_u\mathrm{d}u=\int_{u=0}^{\infty}(\Phi_uT)\,P_{\mathrm{T},u}\mathrm{d}u=\int_{\mathrm{u}=0}^{\infty}A_1\left[\frac{1}{2}\rho u_0^3C_\mathrm{P}(\Phi_uT)\right]\mathrm{d}u \quad (9.77)$$

where A_1 is the swept area of the turbine and u the ambient wind speed (u_0 in Figure 9.6). The average power extracted if the air density is considered constant is

$$\frac{E}{T}=\overline{P}_\mathrm{T}=\frac{\rho A_1}{2}\int_{u=0}^{\infty}\Phi_u u^3C_\mathrm{P}\mathrm{d}u \quad (9.78)$$

The *capacity factor* is the annual average power generated as a proportion of the turbine rated power. So, in principle:

$$\text{Capacity factor}=(\rho A_1/2)\int_{u=0}^{\infty}[\Phi_u u^3\mathrm{C_p}\,\mathrm{d}u]/[(\mathrm{C_p}\rho A_1/2)u_\mathrm{R}{}^3]$$

This integral or summation cannot be evaluated until the dependence of C_P on the upstream wind speed $u=u_0$ is established.

It is usually considered that there are four distinct wind speed regions of operation, see. Figure 9.23:

1 u_0 *less than cut-in speed* u_ci:

$$E_u=0\ \mathit{for}\ u_0<u_\mathrm{ci} \quad (9.79)$$

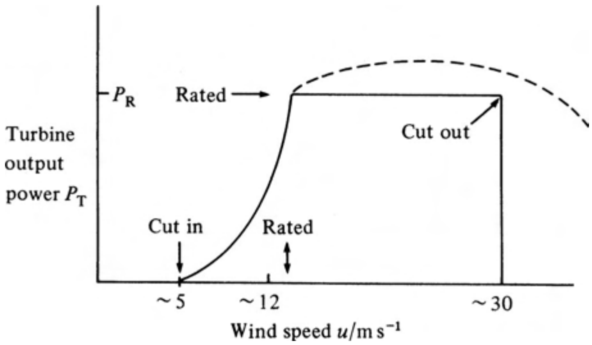

Figure 9.23 Wind turbine operating regions and power performance
_______standard characteristics; requiring exact blade pitch control
_ _ _ _ _actual operating characteristics of many machines, including stall regulation.

2 *u_0 greater than rated speed u_R, but less than cut-out speed u_{co}:*

$$E_u = \Phi_{u>u_R} P_R T \tag{9.80}$$

where P_R is the rated power output (i.e. in this range the turbine is producing constant power P_R).

3 *u_0 greater than cut-out speed u_{co}:*

$$E_u = 0 \; for \; u_0 > u_{co} \tag{9.81}$$

However, in practice many machines do not cut out in high wind speeds because of stall regulation, but continue to operate at greatly reduced efficiency at reasonably large power.

4 *u between u_{ci} and u_R*
The turbine power output P_T increases with u in a way that depends on the operating conditions and type of machine. For many machines

$$P_T \approx a u_0^3 - b P_R \tag{9.82}$$

where a and b are constants.

At cut-in, $P_T = 0$; so

$$u_{ci}^3 = b P_R / a$$

At rated power $P_T = P_R$; so

$$u_R^3 = (1 + b) P_R / a$$

Thus

$$(u_{\text{ci}}/u_{\text{R}})^3 = b/(1+b) \tag{9.83}$$

Hence a and b can be determined in terms of u_{ci}, u_{R} and P_{R}.

In practice, turbines will often be operating in the region between cut-in and rated output, and it is wasteful of energy potential if the machine is unduly limited at large wind speeds. There are two extreme theoretical conditions of operation, see Figure 9.24:

1. *Variable rotor speed for constant tip-speed ratio* λ, *hence constant* C_{P}. This is the most efficient mode of operation and captures the most energy, see Problem 9.13 (and its answer) for details of calculating the energy capture. Variable speed turbines usually cut in at wind speeds less than for constant speed turbines, which also increases energy capture.
2. *Constant (fixed) turbine rotational frequency, hence varying* C_{P}. Although less efficient than variable speed turbines, the use of standard induction generators allows easy grid connection (the small frequency slip of induction generators is not significant, so the machines are described as 'constant speed').

From Figure 9.24, it can be seen that C_{P} can be obtained as a function of unperturbed wind speed u_0, and the turbine power calculated by numerical methods. By operating at constant frequency there is a loss of possible energy extraction. This may be particularly serious if there is a mismatch of optimum performance at larger wind speeds.

In practice, a measured (or estimated) operating power curve of a wind turbine is usually supplied by the maker, in the form of a curve like Figure 9.23 or as a data table, and the term 'capacity factor' is used for the ratio of actual annual average generated power at a site, divided by the generator name-plate rated power.

9.8 Electricity generation

9.8.1 Basics

See Section 16.9 for a basic description of the various types of electrical generator and of electricity networks or 'grids'.

Electricity is an excellent energy vector to transmit the captured mechanical power of a wind turbine. Generation is usually ~95% efficient, and transmission losses should be less than 10%. There are already many designs of wind/electricity systems including a wide range of generators. Future

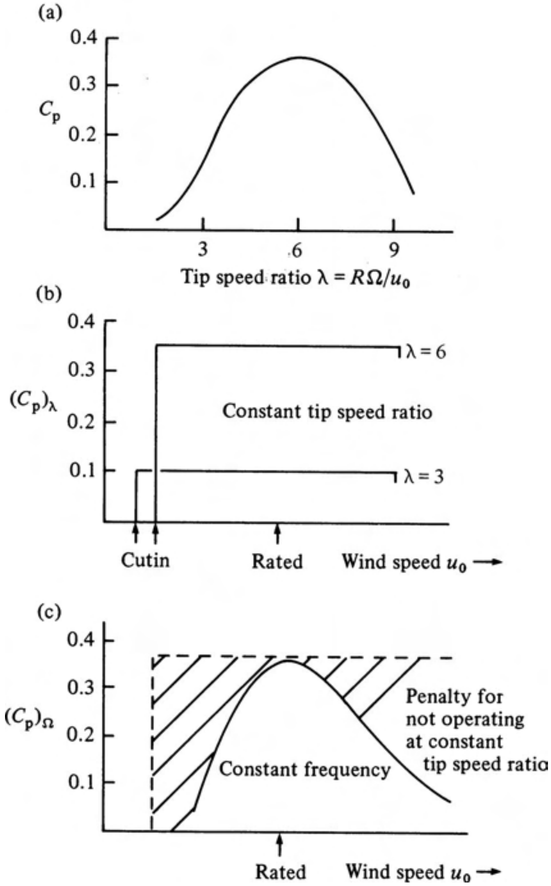

Figure 9.24 Power coefficient C_p: (a) Versus tip-speed ratio. (b) Versus wind speed at constant tip-speed ratio and so variable rotor speed. (c) Versus wind speed at constant turbine frequency, compared with variable speed at tip-speed ratio of 6.

development will certainly produce new and improved designs of generators and control systems as wind-generated power becomes an electrical engineering speciality.

Grid connected turbines and wind farms dispatch power to be integrated with other forms of generation, e.g. thermal power stations. Consumers use power at nearly constant voltage and frequency, as controlled by the grid operators for the power transmission system. However, power from the wind varies significantly and randomly. If the power from wind is no more than about 20% of the total power at one time, then the variations are usually acceptable within the ever changing conditions of the consumer loads.

For stand-alone applications, the frequency and voltage of transmission need not be so standardised, since end-use requirements vary. Heating in particular can accept wide variations in frequency and voltage.

In all applications it will be necessary to match carefully the machine characteristics to the local wind regime. Obviously extended periods of zero or light wind will limit wind-power applications. In particular, sites with an average wind speed less than $5\,\mathrm{m\,s^{-1}}$ usually have unacceptably long periods at which generation would not occur, although water pumping into water storage may still be feasible. Usually, if the annual average wind speed at 10 m height is $5\,\mathrm{m\,s^{-1}}$ or more, electricity generation may be contemplated.

The distinctive features of wind/electricity generating systems are:

1 Wind turbine efficiency is greatest if rotational frequency varies to maintain constant tip-speed ratio, yet electricity generation is most efficient at constant or nearly constant frequency.
2 Mechanical control of a turbine by blade pitch or other mechanical control at powers less than rated increases complexity and expense. An alternative method, usually cheaper and more efficient but seldom done, is to vary the electrical load on the turbine to control the rotational frequency.
3 The optimum rotational frequency of a turbine (its ‘speed’) in a particular wind speed decreases with increase in radius in order to maintain constant tip-speed ratio. Thus only small (~2 m radius) turbines can be coupled directly to conventional four or six pole-pair generators. Larger machines require a gearbox to increase the generator drive frequency or special multipole generators. Gearboxes are relatively expensive and heavy; they require maintenance and can be noisy.
4 The rotor can be decoupled from the load, with the advantage of allowing the rotor to be optimised to the wind. Some developments experimented with a mechanical accumulator (e.g. a weight lifted by hydraulic pressure), but predominantly an electrical method is used. For autonomous systems, chemical batteries provide both this decoupling and longer-term energy storage. For grid connected systems, generated AC electricity may be rectified to DC and then inverted to grid frequency AC; very short term, but useful, ‘rotor inertia’ energy storage occurs, which smoothes wind turbulence. Even the provision of a ‘soft coupling’ using teetered blades, shock absorbers or other mechanisms is useful to reduce electrical spikes and mechanical strain.
5 There are always periods without wind. Thus wind turbines must be linked to energy storage or parallel generating systems, e.g. through the utility power grid, so supplies are to be maintained.

Often, the best wind-power sites are in remote rural, island or marine areas. Energy requirements in such places are distinctive, and almost certainly will not require the intense electrical power of large industrial complexes. In such cases, explicit end-use requirements for controlled electricity (e.g. 240 V/50 Hz or 110 V/60 Hz for lighting, machines and electronics) are likely to be only 5–10% of the total energy requirement for transport, cooking and heat. As wind power experience increases, further developments can be expected so the wind provides affordable energy for heat and transport, in addition to standard electrical uses. Such novel developments occurred first in some remote area power systems, e.g. the Fair Isle system described in Example 9.4 (Section 9.8.4). Moreover, rural grid systems are likely to be 'weak', since they carry relatively low-voltage supplies (e.g. 33 kV) over relatively long distances with complicated inductive and resistive power loss problems. Interfacing a large wind turbine in weak grids is challenging, but certainly not impossible with modern power electronics; indeed the wind power can be used to strengthen the grid supply, for instance by controlling reactive power and voltage.

9.8.2 Classification of electricity systems using wind power

There are three classes of wind turbine electricity system, depending on the relative size of the wind-turbine generator, P_T, and other electricity generators connected in parallel with it, P_G (Table 9.4).

Class A: *wind turbine capacity dominant*, $P_T \geq 5P_G$
Usually this will be a single autonomous stand-alone machine without any form of grid linking. Other generators are not expected. For electricity supply, a battery is necessary to stabilize the voltage and store the electricity. For remote communication sites, household lighting, marine lights etc.,

Table 9.4 Classes of wind turbine electricity systems

Class	*A*	*B*	*C*
P_T: wind turbine generator P_G: other generator Capacity	$P_T \gg P_G$	$P_T \sim P_G$	$P_T \ll P_G$
Example	Autonomous	Wind/diesel	Grid embedded
Control modes	(a) Blade pitch (b) Load matching	(a) Wind or diesel separately (b) Wind and diesel together	(a) Direct induction generator (b) To DC then AC (c) Increased slip induction generator

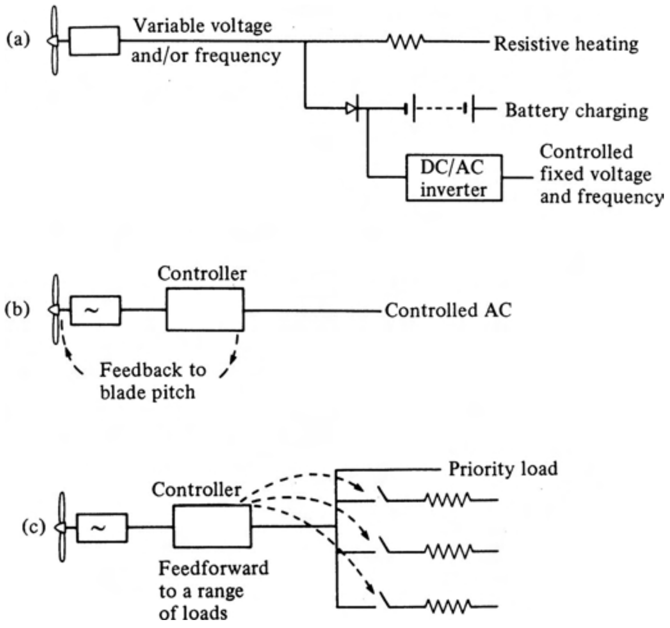

Figure 9.25 Some supply options with the wind turbine the dominant supply.

$P_T \leq 2\,\text{kW}$. For full household supplies, including heat, $P_T \sim 10\,\text{kW}$. Wind turbines of large capacity are likely to have standby generation of class B.

Control options have been discussed in Section 1.5.4 and are of extreme importance for efficient cost-effective systems, Figure 9.25. One choice is to have very little control so the output is of variable voltage (and, if AC, frequency) for use for direct resistive heating or for rectified power, Figure 9.25(a). There are many situations where such a supply will be useful. The relatively small amount of power that usually has to be controlled at, say, 240 V/50 Hz or 110 V/60 Hz can be obtained from batteries by inverters. Thus the high quality controlled electricity is obtained 'piggy-back' on the supply of less quality, and can be costed only against the marginal extras of battery and inverter.

However, it may be preferred to have the electricity at controlled frequency. There are two extreme options for this:

1 *Mechanical control of the turbine blades.* As the wind changes speed, the pitch of the blades or blade tips is adjusted to control the frequency of turbine rotation, Figure 9.25(b). The disadvantages are that power in the wind is 'spilt' and therefore wasted (see Section 1.5) and the control method may be expensive and unreliable.

2 *Load control*. As the wind changes speed, the electrical load is changed by rapid switching, so the turbine frequency is controlled, Figure 9.25(c). This method makes greater use of the power in the wind by optimising tip-speed ratio λ. Moreover local control by modern electronic methods is cheaper and more reliable than control of mechanical components exposed in adverse environments.

Permanent magnet multipole generators are common for small machines. DC systems can be smoothed and the energy stored in batteries. AC systems may have synchronous generators producing uncontrolled output for heat, or controlled output by mechanical or load control. AC induction generators can be self-excited with a capacitor bank to earth, or may operate with an idling synchronous generator as a compensator (see Section 16.9.3 for further discussion of generator types).

Class B: *wind turbine capacity* ≈ *other generator capacity*, $P_T \approx P_G$
This is a common feature of remote area, small grid systems. We shall first assume that the 'other generator' of capacity P_G is powered by a diesel engine, perhaps fuelled by biodiesel. The principal purpose of the wind turbine is then to be a saver of the fuel. The diesel generator will be the only supply at windless periods and will perhaps augment the wind turbine at periods of weak wind. There are two extreme modes of operation:

1 *Single-mode electricity supply distribution*. With a single set of distribution cables (usually a three-phase supply that takes single phase to domestic dwellings), the system must operate in a single mode at fixed voltage for 240 V or 110 V related use, Figure 9.26(a). A 24 h maintained

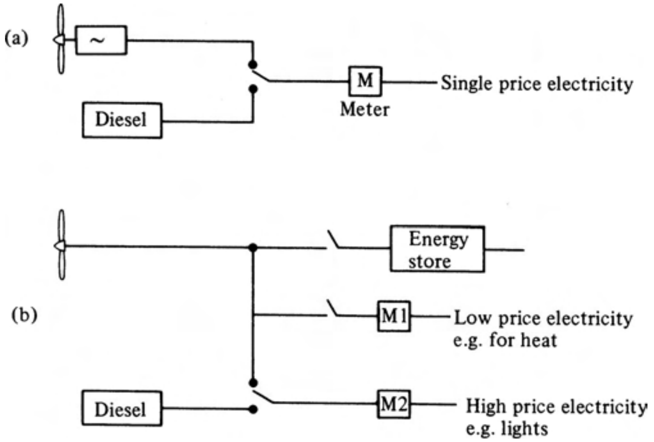

Figure 9.26 Wind/diesel supply modes. (a) Single mode. (b) Multiple mode.

supply without load management control will still depend heavily (at least 50% usually) on diesel generation since wind is often not available. The diesel is either kept running continuously (frequently on light load, even when the wind power is available) or switched off when the wind power is sufficient. In practice a large amount (sometimes over 70%) of wind-generated power has to be dumped into outside resistor banks owing to the mismatch of supply and demand in windy conditions.

2 *Multiple-mode distribution.* The aim is to use all wind-generated power by offering cheap electricity for many uses in windy conditions, Figure 9.26(b). As the wind speed decreases, the cheaper serviced loads are automatically switched off to decrease the demand, and vice versa. The same system can be used to control the rotation of the wind turbine. When no wind power is available, only the loads on the expensive supply are enabled for supply by the diesel generator. The pragmatic economic advantage of successful multiple-mode operation is that the full capital value of the wind machine is used at all times, and since the initial power in the wind is free, the maximum benefit is obtained. It is also advantageous in using less fuel with the abatement of pollution and noise.

Class C: grid linked, wind turbine embedded in a large system, $P_T \leq 0.2P_G$
This is the most common arrangement for large (~3 MW), medium (~250 kW) and small (~50 kW) machines where a public utility or other large capacity grid is available. In recent years, institutional factors have led to the bulk of new wind-power capacity being in 'wind farms', in which a number (10–1000) of turbines in a group are all feeding into the grid (see next section and Chapter 17). For smaller systems, the owner of the machine may use the wind power directly and sell (export) any excess to the grid, with electricity purchased (imported) from the grid at periods of weak or no wind (Figure 9.27).

The cheapest type of generator is an induction generator connected directly to the grid. The turbine has to operate at nearly constant frequency,

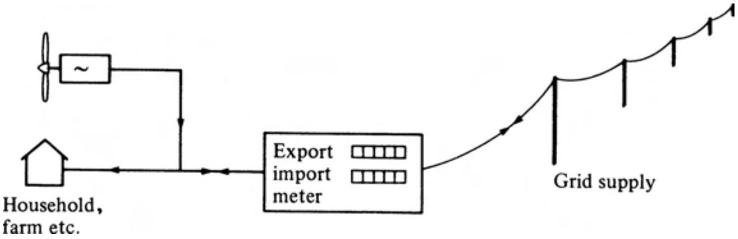

Figure 9.27 Grid linked wind turbine slaved in a large system.

within a maximum slip usually less than 5% ahead of the mains-related frequency; this is usually called 'fixed speed'. In weak wind, there is an automatic cut-out to prevent motoring. The disadvantage of a directly coupled induction generator is that the turbine frequency cannot change sufficiently to maintain even approximate constant tip-speed ratio.

However, there are several ways in which the system can be made to produce electricity at fairly constant frequency while allowing variable turbine frequency. They include: (1) multiple (usually two) combination windings in an induction generator to connect more pole pairs in weak winds for smaller rotational frequency; (2) some intermediate scale machines use two generators in the same nacelle, say 5 kW and 22 kW, for automatic connected to a two-speed gearbox in light and strong winds; (3) using a variable speed generator and rectifying its output to direct current and then producing the prescribed alternating current mains frequency with an inverter; and (4) increasing the effective slip on an induction generator by active change of the current and phase in the generator's rotor, e.g. in a doubly-fed induction generator; this requires external electrical connection to the rotor winding via slip rings and brushes.

9.8.3 Electricity generation for utility grids: wind farms

Commercial wind turbines are a 'mainstream' form of power generation into grid distribution and transmission networks. Machines of megawatt capacity have operated successfully for many years. Multiple numbers of machines arrayed on 'wind farms' (typically with 10–100 turbines) make convenient and manageable units. Grouping machines in this way allows savings (10–20%) in the construction costs (e.g. getting specialised cranes etc. on site), grid connection (fewer step-up transformers required), management and maintenance. Wind farms are most likely in countries where there is (1) a commitment to sustainable, low-carbon, energy supplies, (2) previous strong dependence on brown energy and (3) open areas with an average wind speed at 10 m height $> 6\,\mathrm{m\,s^{-1}}$.

Since wind speeds are usually larger offshore than onshore, except in mountainous regions, it can be beneficial to locate wind farms up to several kilometre offshore. This approach is particularly attractive in marine countries where potential onshore sites are limited by dense population, visual intrusion, and failure to gain planning permissions.

Because the output from wind turbine power is less predictable than that from a conventional system (fossil, nuclear or hydro), adding a wind turbine rated at 1 MW capacity to a grid is not equivalent to adding 1 MW capacity from a 'brown' source. In general the site-dependent annual capacity factor of a wind turbine is 20–35%, whereas a thermal power station is about 70–90%. Yet, not all 'brown' sources are equivalent, e.g. nuclear power is suitable only for base load, whereas gas turbines are best for rapid response to peak demands. Energy economists describe this in terms of *capacity credit*, the power rating of conventional plant that is displaced by the installation of wind power or other renewable energy. Theoretical studies by numerous utilities have predicted that 1000 MW (rated) wind power has a capacity credit of 250–400 MW (Milborrow 2001). The larger figure corresponds to sites of larger average wind speeds, since these have more extended periods of wind. If the wind power comes from a diversity of sites, there is less chance of them all having reduced output at the same time, and so the predicted capacity credit is larger – in some cases arguably close to 100%. A related opinion is that wind power requires the construction of back-up power stations. Therefore a utility network always has to have reserve generating capacity available for all forms of generation, especially since large power stations fail at times, and so to the authors' knowledge no additional back-up power has yet been needed or constructed due to extra wind power capacity.

9.8.4 Individual machines and integrated systems

Most wind turbine capacity is associated with commercial wind farms for grid connected power, and therefore large machines (~ 3 MW) are the most common now. However small machines of capacity between about 50 W and1 kW are common for boats, holiday caravans and houses, small power public service (e.g. rural bus shelters) and small meteorological and other measurement sites. Slightly larger, but still 'small' are 5–100 kW wind turbines installed for household, farm and institutional use. Cost-effective operation is most likely in locations where other energy supplies are expensive (e.g. oil) and grid electricity not available. However, where there is a grid and if excess electricity can be sold to a utility grid at a price of at least half the buying price, grid connection is no discouragement for wind power projects.

The principles of renewable energy supply, developed in Chapter 1, indicate that the renewables technology has to operate within quite different constraints than have fossil fuel and nuclear sources. The dispersed and highly fluctuating nature of wind attracts radically different approaches than those used for steady intensive sources. In particular, there is scope for adaptation of the end-uses of the wind-generated power so that the load responds to the changing supply, and energy storage is incorporated, see Chapter 16. The multi-mode system at Fair Isle (Example 9.4 and Figure 1.6) illustrates what can be achieved by taking an integrated whole-system approach, covering both supply and use of energy. Such an approach is possible, but very uncommon, on much larger systems.

Example 9.4 Fair Isle multimode wind power system

Fair Isle is an isolated Scottish island in the North Sea between mainland Shetland and Orkney. The population of 70 is well established and progressive within the limits of the harsh yet beautiful environment. Previously the people depended on coal and oil for heat, petroleum for vehicles, and diesel for electricity generation. The electricity co-operative installed first a 50 kW rated-capacity wind turbine that operates in the persistent winds, of average speed 8–9 $\mathrm{m\,s^{-1}}$. The control system was mentioned in Chapter 1, see Figure 1.6. Lighting and power outlets receive electricity metered at larger price, and a reduced price controlled-supply is available (wind permitting) for hard-wired comfort heat and water heating, see Section 1.5.4. At the frequent periods of excessive wind power, further heat is available, e.g. for growing food in a glasshouse or for a small swimming pool. An electric vehicle was charged from the system to include transport as an end-use. Despite the strong winds, the total generating capacity is small for the population served, and acceptable standards are only possible because the houses are well insulated, careful energy strategies are maintained and sophisticated reliable control systems are incorporated.

9.9 Mechanical power

Historically the mechanical energy in the wind has been harnessed predominantly for transport with sailing ships, for milling grain and for pumping water. These uses still continue and may increase again in the future. This section briefly discusses those systems, bearing in mind that electricity can be an intermediate energy vector for such mechanical uses.

9.9.1 Sea transport

The old square rigged sailing ships operated by drag forces and were inefficient. Modern racing yachts, with a subsurface keel harnessing lift forces, are much more efficient and can sail faster than the wind. Some developments to modern cargo ships have used fixed sails set by mechanical drives.

9.9.2 Grain milling

The traditional windmill (commonly described as a Dutch windmill) has been eclipsed by engine- or electrical-driven machines. It is unlikely that the intermittent nature of wind over land will ever be again suitable for commercial milling in direct mechanical systems.

9.9.3 Water pumping

Pumped water can be stored in tanks and reservoirs or absorbed in the ground. This capacitor-like property gives smoothing to the intermittent wind source, and makes wind-powered pumping economic. Farm scale pumps to about 10 kW maximum power are common in many countries including Argentina, Australia and the United States. The water is used mostly for cattle, irrigation or drainage. Continuity of supply is important, so large solidity multi-blade turbines are suitable, having large initial torque in weak winds. The small rotational speed is not a handicap for direct mechanical action. The traditional cylinder pump with a fixed action, Figure 9.28, is simple and reliable. At best, however, the delivered power is proportional to turbine rotational frequency ($P' \propto \Omega$), whereas at constant tip-speed ratio the power at the turbine is proportional to $\Omega^3 (P_T \propto \Omega^3)$: Therefore the efficiency P'/P_T drops as $1/\Omega^2$. Improved pumps that match the wind turbine characteristics and maintain simplicity of operation are important for more efficient water pumping. Since water is usually available at low locations, and wind increases with height, it is often sensible to have an electricity-generating wind turbine placed on a hill operating an electric pump placed at the nearby water supply.

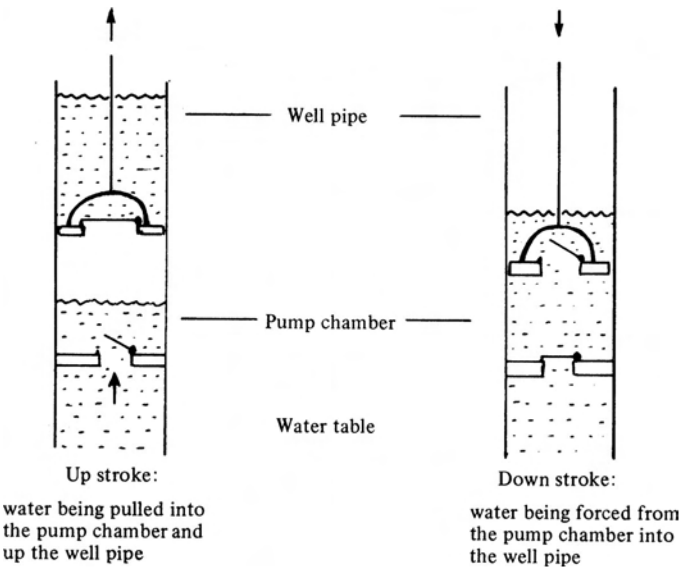

Figure 9.28 Positive displacement water pump. The shaft would be connected to the rotating crankshaft of the wind turbine.

9.9.4 Heat production

The direct dissipation of the mechanical power from a wind turbine (e.g. by paddlewheel systems) produces heat with 100% efficiency. However, electrical generators are so common and efficient that it is certain that electricity will be favoured as the intermediate energy vector to electrically powered heating.

9.10 Social and environmental considerations

Some of the potentially best locations for wind-power are in areas of natural beauty, such as coastlines, high ground and mountain passes. Proposals to use such locations usually attract opposition with arguments of loss of visual amenity, irritating acoustic noise and bird strikes. Similar objections have been raised to wind farms on farmland. Manufacturers of modern wind turbines responded by having architects influence the shape of towers and nacelles, by making the machines (especially the gear boxes) much quieter and by employing ecologists to advise on sites with least adverse and most advantageous impact on animals and plants. Some early wind farms in western USA were located in mountain passes on bird migration paths of birds and consequently bird-kill became a concern. However in most locations, birds can and do fly around the turbines without hazard. In general, since normal ecological and farming processes can continue underneath the spread of the rotor wherever it is, as in Figure 9.29, there is no environmental impact, other than on human opinion and perhaps on certain types of bird. The impacts on access for low flying military planes, on radar generally and on TV and communication channels have also to be considered.

There is a definite danger that wind power development will be pushed only by those with technical understanding. This is a definite mistake, since there should be appreciation of ecology, aesthetics, cultural heritage and public perceptions. These ‘other’ aspects are well reviewed in Pasqualetti *et al.* (2002) *Wind Power in View*; readers are strongly recommended to consider the lateral perceptions of this study.

In most countries, wind farm developers have to obtain local planning permission before installing a wind farm. Consequently the process of preparing for an application has become comprehensive and professional. Simulation software is used to give a dynamic visual impression of the wind farm from all viewpoints, in-depth bird and other ecological studies are funded, acoustic noise is predicted in the vicinity, effect on roads is studied, local benefit may be publicly offered (e.g. cheaper electricity supplies, donations to schools) and many other issues considered. If an application is refused, then appeals may be made. All these procedures are necessary, but time-consuming and expensive.

Yet the final outcome is that national and world wind power capacities are increasing, carbon and other emissions are being abated, the technology

Figure 9.29 A wind farm in Minnesota, USA, with agricultural activity continuing underneath. [Photo by Warren Gretz, courtesy of [US] National Renewable Energy Laboratory.]

is improving and many adverse impacts, per unit of generated output, are decreasing.

Problems

9.1 From (9.16) the fraction of power extracted from the wind is the power coefficient $C_P = 4a(1-a)^2$. By differentiating with respect to a, show that the maximum value of C_P is 16/27 when $a = 1/3$.

9.2 The calculation of power coefficient C_P by linear momentum theory (Section 9.3.1) can proceed in terms of $b = u_2/u_0$. Show that (a) $C_P = (1-b^2)(1+b)/2$, (b) C_P is a maximum at 16/27 when $b = 1/3$, (c) $a = (1-b)/2$ where $a = (u_0 - u_1)/u_0$, and (d) the drag coefficient $= (1-b^2)$.

9.3 a By considering the ratio of the areas A_0 and A_1 of Figure 9.6, show that the optimum power extraction (according to linear momentum theory) per unit of area A_0 is 8/9 of the incident power in the wind.